
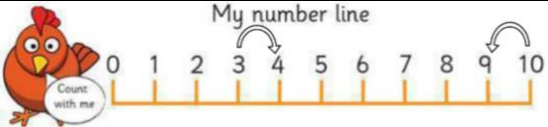


Objective and strategy	Concrete	Pictorial	Abstract	<h1 style="writing-mode: vertical-rl; text-orientation: mixed;">Y1 ADDITION +</h1>
Combining two parts to make a whole: part-whole model	<p>Use part part whole model. Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$ $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.	
Starting at the bigger number and counting on	<p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>I can start on the bigger number and count on.</p>	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.	
Regrouping to make 10. This is an essential skill for column addition later.	<p>Start with the bigger number and use the smaller number to make 10. Use ten frames.</p>	<p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p>	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?	

<p>Represent & use number bonds and related subtraction facts within 20</p>	 <p>2 more than 5.</p>	<p>My number line</p>  <p>$3 + 1 = 4$ $10 - 1 = 9$</p>	<p>Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'</p>	
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Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line*

Key skills for addition at Y1:

- Read and write numbers to 100 in numerals, incl. 1–20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$ Model using dienes and bead strings	 Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	 $\square + \square = \square$ $\square + \square = \square$	 Children draw representations of H, T and O	$3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$



Objective & Strategy	Concrete	Pictorial	Abstract				
<p>Add a two digit number and ones</p>	<p>$17 + 5 = 22$ Use ten frame to make 'magic ten'</p> <p>Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$</p>	<p>Use part part whole and number line to model.</p> <p>$17 + 5 = 22$</p>	<p>$17 + 5 = 22$</p> <p>Explore related facts</p> <p>$17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="2" style="text-align: center;">22</td></tr> <tr><td style="width: 50%;">17</td><td style="width: 50%;">5</td></tr> </table>	22		17	5
22							
17	5						
<p>Add a 2 digit number and tens</p>	<p>$25 + 10 = 35$</p> <p>Explore that the ones digit does not change</p>	<p>$27 + 30$</p> <p>27 37 47 57</p>	<p>$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$</p>				
<p>Add two 2-digit numbers</p>	<p>Model using dienes, place value counters and numicon</p>	<p>$45 + 22$</p> <p>45 46 47 57 67</p>	<p>$34 + 13$ $4 + 3 = 7$ $30 + 10 = 40$ $40 + 7 = 47$</p>				
<p>Add three 1-digit numbers</p>	<p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	<p>Regroup and draw representation.</p> <p>$4 + 7 + 6 = 15$</p>	<p>$4 + 7 + 6 = 10 + 7$ $= 17$</p> <p>Combine the two numbers that make/bridge ten then add on the third.</p>				

Y2 ADDITION +

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line* , **sum, tens, units, partition, addition, column, tens boundary**

Key skills for addition at Y2:

- Add a 2-digit number and ones (e.g. $27 + 6$)
- Add a 2-digit number and tens (e.g. $23 + 40$)
- Add pairs of 2-digit numbers (e.g. $35 + 47$)
- Add three single-digit numbers (e.g. $5 + 9 + 7$)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 ($30 + 70$ etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using $<$ $>$ and $=$ signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.



Y3 ADDITION +

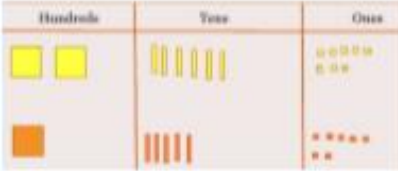
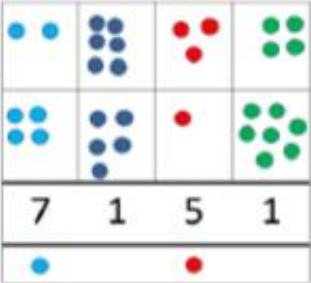
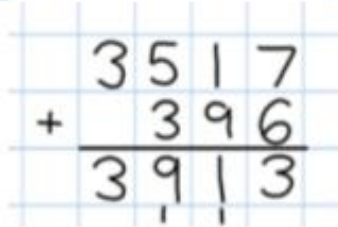
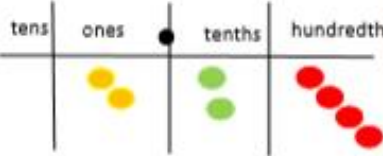
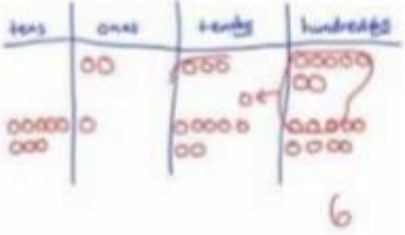
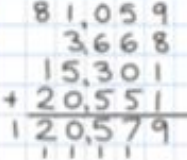
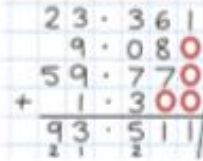
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p>	<p>Model using Dienes or numicon</p> <p>Add together the ones first, then the tens.</p> <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p>	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p>	<p>Exchange ten ones for a ten. Model using numicon and pv counters.</p>	<p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p>	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$



Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact*

Key skills for addition at Y3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- Add a three-digit number and ones mentally ($175 + 8$)
- Add a three-digit number and tens mentally ($249 + 50$)
- Add a three-digit number and hundreds mentally ($381 + 400$)
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Y4—add numbers with up to 4 digits</p>	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures.</p>
<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	<p>72.8 + 54.6 ----- 127.4</p> <p>11 £23.59 + £7.55 ----- £31.14</p>
<p>Y6—add several numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>As Y5</p>	<p>As Y5</p>	 <p>Insert zeros for place holders.</p> 

Y4-6 ADDITION +

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, **thousands, hundreds, digits, inverse***

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods

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Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths***

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

LIVINGSTONE

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths*

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

LIVINGSTONE

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Part Whole model</p>	<p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	<p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	<p>$14 - 9$</p> <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>$13 - 7$</p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>$16 - 8$</p> <p>How many do we take off first to get to 10? How many left to take off?</p>
<p>Bar model</p>	$5 - 2 = 3$		$10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$

Y1 SUBTRACTION -

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Y1:



- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.
- Use **reasoning** skills to solve one step problems and missing number problems: $7 = \square - 9$
- Use reasoning skills to solve one step subtraction problems in context of money and measures.
- Begin to recognise addition and subtraction are inverse of each other.

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Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	<p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	$20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$ <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p> $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	$34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	<p>'counting on' to find 'difference'</p> <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$
		$46 - 22$	

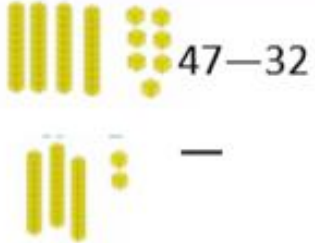

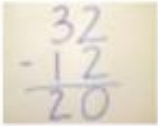

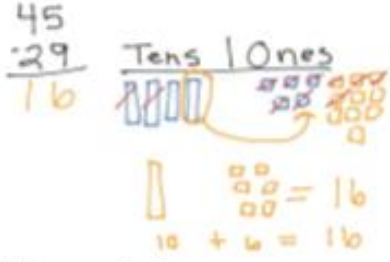


Y2 SUBTRACTION -

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, tens, units

Key skills for subtraction at Y2:

- Recognise the place value in each digit of a 2 digit number
- Recall and use subtraction facts up to 20 and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two digit numbers and ones, a 2 digit number and tens and 2 two digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Read and write numbers up to 100 in numerals and words.
- Use **reasoning skills** to solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.

Y3 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p>Use base 10 or Numicon to model</p>	 <p>Darw representations to support understanding</p>	<p>$47 - 24 = 23$</p> $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, tens, units, exchange, decrease, hundreds, value, digit

Key skills for subtraction at Y3:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10
- Read and write numbers up to 1000 in numerals and words.
- **Reasoning:** Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p> <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	<p>Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	<p>Use zeros for place-holders.</p>
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</p>			

Y4-6 SUBTRACTION -

Y4

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, tens, units, exchange, decrease, hundreds, value, digit, inverse

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 100
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- **Reasoning** :Solve addition /subtraction 2-step problems, choosing which operations / methods to use.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number; round any number to the nearest 10, 100 or 1000 and solve number and practical problems that involve the above.

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Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is?, difference, count on, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal, decimal point

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly larger numbers.
- Use rounding and estimation to check answers in a range of contexts.
- **Reasoning:** Solve addition and subtraction multi-step problems in a range of contexts deciding which methods to use and why.
- **Reasoning:** Solve problems with decimals up to 3 decimal places.
- **Reasoning:** Solve problems with all four operations in range of contexts and understand meaning of = sign.
- Read, write and compare numbers up to 1,000,000 and know the value of each digit.
- Count forwards and backwards in steps of power 10 for any given number up to 1,000,000
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000
- Use patterns of similar calculations: $19 - 7 = 12$ $1.9 - 0.7 = 1.2$

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal, decimal point

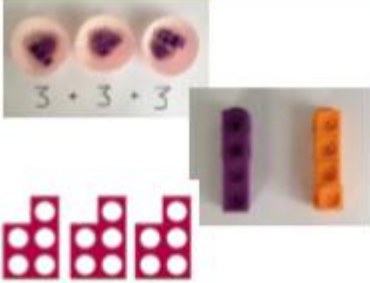
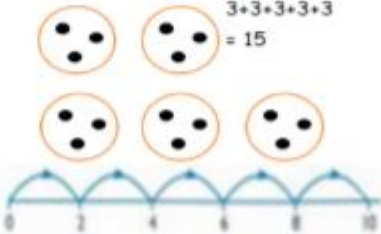

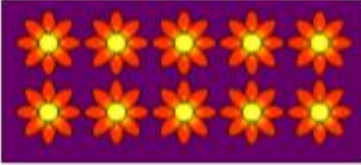
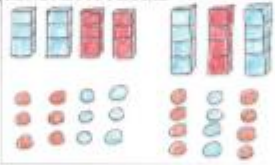
Key skills for subtraction at Y6:

- **Reasoning:** Solve addition and subtraction multi-step problems in context, deciding what calculations and methods to use and why.
- Read, write and compare numbers up to 10 million and know the value of each digit.
- Round any whole number with a greater degree of accuracy
- Use negative numbers in context and calculate intervals
- Consider a range of mental strategies, jottings and written methods before choosing how to calculate.

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Y1 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p> <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p>	<p>Partition a number and then double each part before recombining it back together.</p> <p>16 10 6 $\downarrow \times 2$ $\downarrow \times 2$ $20 + 12 = 32$</p>
<p>Counting in multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p>	<p>Children make representations to show counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
<p>Making equal groups and counting the total</p>	<p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p>

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	$3 \times 2 = 6$ $2 \times 5 = 10$

Y1 MULTIPLICATION X

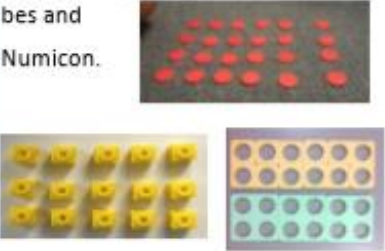

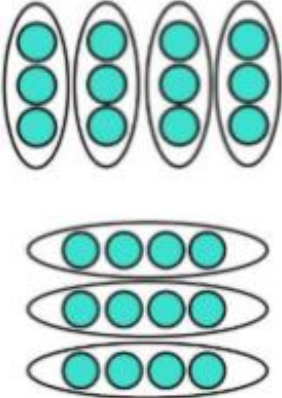


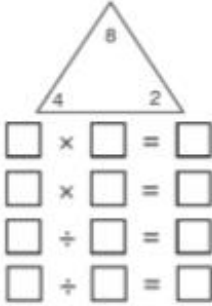
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Yr 1:

- Count in multiples of 2, 5 and 10.
- **Reasoning:** Solve one step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

Y2 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Model doubling using dienes and PV counters.</p> <p>$40 + 12 = 52$</p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p> <p>$20 + 12 = 32$</p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p> <p>$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p>	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> <p>$4 \times 3 = \square$</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p>$4 \times 3 = \square$</p>

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p> 	<p>$12 = 3 \times 4$ $12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>			<p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>

Y2 MULTIPLICATION X

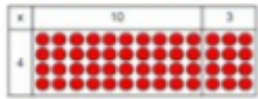
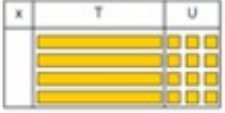

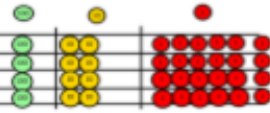
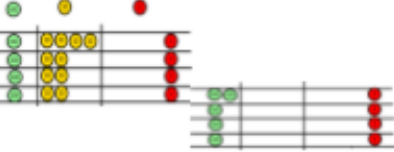
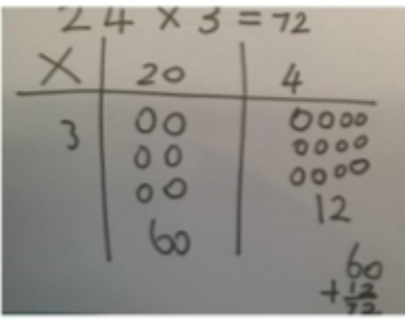
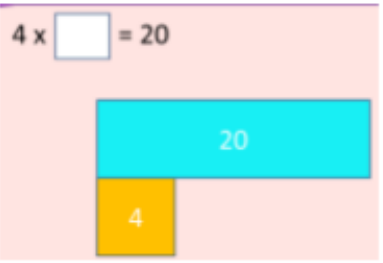
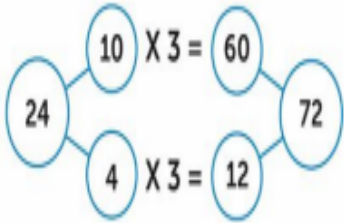
Y2

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, repeated addition, column, row, commutative, sets of, equal groups, times as big as. Once, twice, three times.

Key Skills for multiplication:

- Count in steps of 2, 3, and 5 from 0 and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Write and calculate number statements using the \times and $=$ signs
- Show that multiplication can be done in any order (commutative)
- **Reasoning:** Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods and multiplication facts.
- Pupils use a variety of language to describe multiplication

Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Grid method</p>	<p>Show the links with arrays to first introduce the grid method</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculators 4 x 126</p> <p>Fill each row with 126</p>  <p>Calculators 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> 	

Y3 MULTIPLICATION X

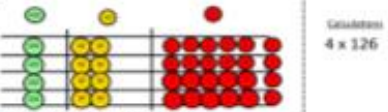
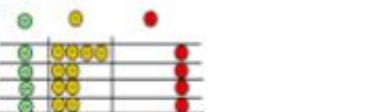
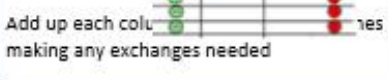
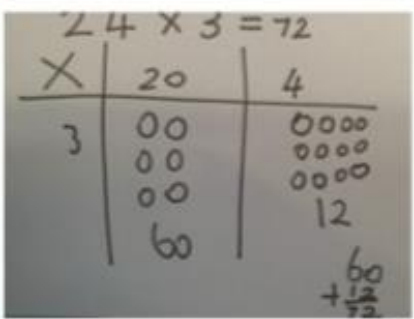
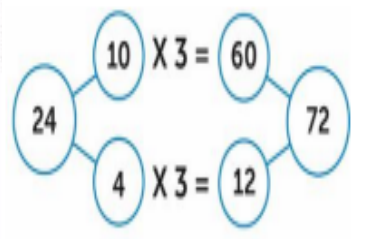
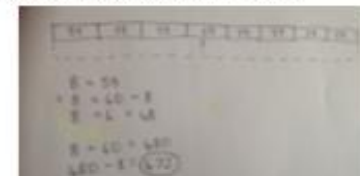
Y3

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, repeated addition, column, sets of, equal groups, times, partition, grid method, multiple, product, tens, units, value

Key Skills for multiplication:

- Recall and use multiplication facts for 2, 3, 4, 5, 8, 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using multiplication tables they know, including $TU \times U$ drawing upon mental methods and move to formal written methods.
- **Reasoning:** Solve problems including missing number problems.
- Develop mental methods knowing multiplication can be done in any order. 7×4 can be done using $(5 \times 4) + (2 \times 4)$

Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract																													
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Fill each row with 126</p>  <p>Add up each column making any exchanges needed</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 																														
<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="465 925 739 1244"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3 blocks</td> <td>2 blocks</td> <td>1 block</td> </tr> <tr> <td>3 blocks</td> <td>2 blocks</td> <td>1 block</td> </tr> <tr> <td>3 blocks</td> <td>2 blocks</td> <td>1 block</td> </tr> <tr> <td>3 blocks</td> <td>2 blocks</td> <td>1 block</td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3 blocks	2 blocks	1 block	3 blocks	2 blocks	1 block	3 blocks	2 blocks	1 block	3 blocks	2 blocks	1 block	<table border="1" data-bbox="940 829 1254 909"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	<table data-bbox="1344 813 1590 1133"> <tr><td>327</td></tr> <tr><td>x 4</td></tr> <tr><td>1308</td></tr> </table> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1344 1149 1545 1292"> <tr><td>327</td></tr> <tr><td>x 4</td></tr> <tr><td>1308</td></tr> </table>	327	x 4	1308	327	x 4	1308
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Y4 MULTIPLICATION X

Y4

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, repeated addition, column, sets of, equal groups, times, partition, grid method, multiple, product, tens, units, value

Key Skills for multiplication:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall all multiplication facts 12 x 12
- Recognise inverse operation for multiplication and division
- Recognise place value of digits in a 4 digit number
- Use place value facts to multiply mentally
- Use commutativity law for mental strategies: $2 \times 6 \times 5 = 10 \times 6$ $39 \times 7 = 30 \times 7 + 9 \times 7$
- **Reasoning:** Solve problems with increasing complexity in range of contexts.

Livingstone

Y5-6 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	<p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p>		<p>This will lead to a compact method.</p>
<p>Column multiplication</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	<p>Continue to use bar modelling to support problem solving</p>	<p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3)</p> <p>18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p>


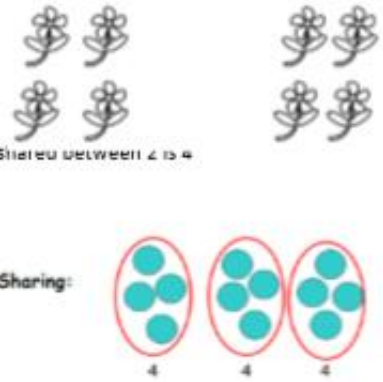
Y5/6

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, repeated addition, column, sets of, equal groups, times, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short / long multiplication

Key Skills for multiplication:

- Use related facts for doubling and halving: double or halve the most significant digit first.
- Multiply 25 by multiplying by 100 and dividing by 4.
- Use factors; 9×18 $9 \times (6 \times 3)$
- Use commutative law 14×12 $(2 \times 7) \times 12$ $2 \times (7 \times 12)$ 2×84
- Identify multiples and factors and use vocab for prime numbers and prime factors
- Recognise all prime, square and cubed numbers up to 100.
- **Reasoning:** Solve problems involving multiplication and factors and prime numbers
- **Reasoning:** Solve problems involving scaling up of simple fractions.

Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p><i>Use Gordon ITPs for modelling</i></p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 Shared between 2 is 4</p> <p>Sharing:</p> <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is</p> <p>4</p>

Y1

DIVISION

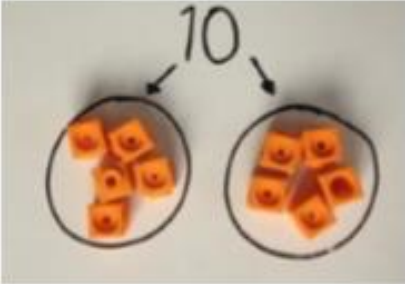




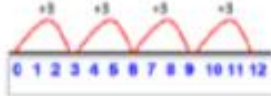


÷

Key Vocabulary: share, share equally, one each, two each., group, groups, lots of, array

Key skills for division at Y1:

- **Reasoning:** Solve one step problems involving multiplication and division, by calculating the answer using the concrete objects, pictures and arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand division and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns and counting in twos, fives and tens.

Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ <p>Children use bar modelling to show and support understanding.</p>  $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping   $12 \div 3 = 4$ <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

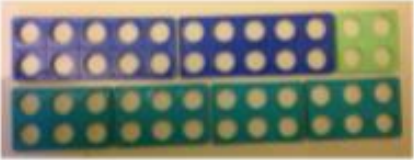
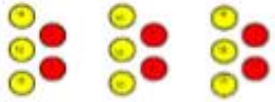


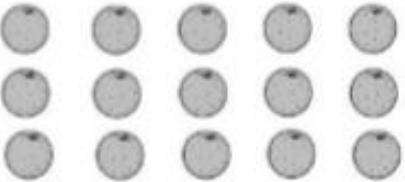
Y2 DIVISION ÷

Key Vocabulary: share, share equally, one each, two each., group, groups, lots of, array, divide, divided by, divided into, division, grouping, numberline, left, left over

Key skills for division at Y2:

- Count on in steps of 2, 3 and 5 from 0.
- Recall and use multiplication facts for the 2, 5 and 10 times table, including recognition of odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div , and $=$ signs.
- Show can be done in any order whereas division can not.
- **Reasoning:** Solve problems involving multiplication and division using concrete materials, arrays and mental methods; include problems in different contexts.

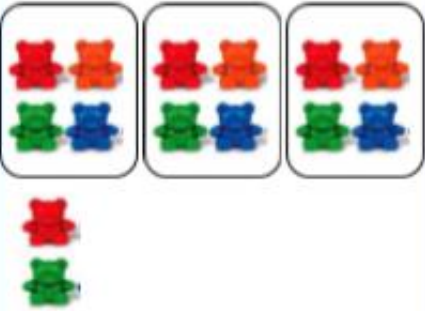


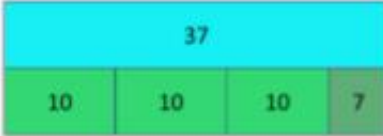
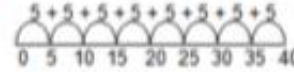
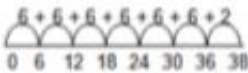
Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Y3 DIVISION ÷



Livingstone

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division with remainders.</p>	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p>Example without remainder: $40 \div 5$ Ask "How many 5s in 40?" $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 \text{ fives}$</p>  <p>Example with remainder: $38 \div 6$</p>  <p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p>	<p>Complete written divisions and show the remainder using r.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <p>↑ ↑ ↑ ↑ dividend divisor quotient remainder</p>

Y3 DIVISION ÷

Key Vocabulary: share, share equally, one each, two each.., group, groups, lots of, array, divide, divided by, divided into, division, grouping, numberline, left, left over, inverse, short division, exchange, multiple

Key skills for division at Y3:

Key Skills for multiplication:

- Recall multiplication facts 2, 3, 4, 5, 8 and 10.
- Write mathematical statements for multiplication and division using tables they know including for 2 digit numbers times 1 digit numbers, using mental and formal methods.
- **Reasoning:** Solve problems in contexts, and including missing number problems.
- Develop efficient methods: $30 \times 2 = 60$ so $60 \div 3 = 20$
- Pupils develop reliable formal written method for division and progress to formal method.

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p> <p>Use place value counters to divide using the bus stop method alongside</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p> <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \\ 7 \\ \underline{7} \\ 0 \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 3 \\ \underline{3} \\ 0 \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{14} \\ 21 \\ \underline{21} \\ 0 \end{array}$

Y4-6

DIVISION

÷

Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y6
DIVISION ÷

Long Division

Y6 DIVISION ÷

Step 1 continued...

$$\begin{array}{r}
 \text{h t o} \\
 061 \\
 4 \overline{) 247} \\
 \underline{-4} \\
 3
 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r}
 \text{th h t o} \\
 0402 \\
 4 \overline{) 1609} \\
 \underline{-8} \\
 1
 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

<p>Y4</p>	<p>Key Vocabulary: share, share equally, one each, two each., group, groups, lots of, array, divide, divided by, divided into, division, grouping, numberline, left, left over, inverse, short division, exchange, multiple divisible by, factor</p>
<p>Key skills for division at Y4:</p>	<ul style="list-style-type: none"> - Recall all facts up to 12×12. - Use place value and derived facts to multiply and divide mentally, including multiplying and dividing by 10, 100 and 1. - Pupils become fluent in the formal written methods of short division. - Pupils practise mental methods extending to 3 digits: $200 \times 3 = 600$ so $600 \div 3 = 200$ - Reasoning: Solve division problems in context deciding which methods to use and why.
<p>Y5/6</p>	<p>Key Vocabulary: share, share equally, one each, two each., group, groups, lots of, array, divide, divided by, divided into, division, grouping, numberline, left, left over, inverse, short division, exchange, multiple divisible by, factor, prime number and factors, composite</p>
<p>Key skills for division at Y5/6:</p>	<ul style="list-style-type: none"> - Divide 4 digits with single or 2 digit numbers and express remainders as fraction / decimal - Round up down in the context of the problem - Perform mental calculations including mixed operations and larger numbers - Identify common factors, common multiples and prime numbers - Solve problems using all 4 operation - Reasoning: Estimate and check answers for accuracy in the context of the problem - Use written methods in cases where the answer has up to 2 decimal places.



Livingstone